Quple
math'n'magic of gamer acquisition

PREDICTIVE BIDDING MODEL

For Programmatic Media Buying Optimization Problem

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We employ **artificial intelligence**
to create next-gen products
for **game marketing**

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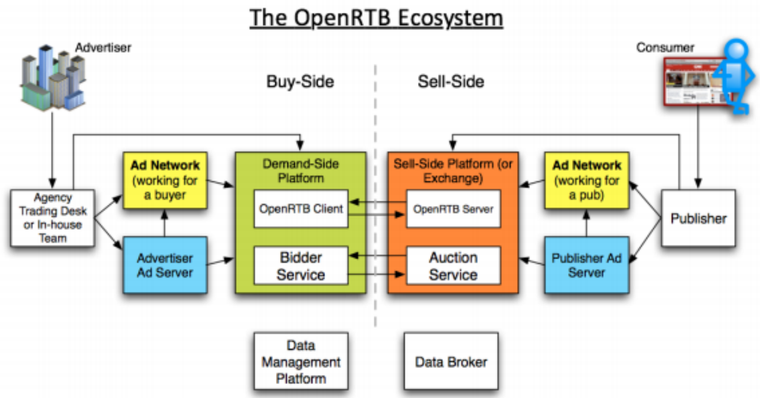
The OpenRTB Ecosystem
Advertiser
Consumer
Buy-Side
Sell-Side
Agency Trading Desk or In-house Team
Ad Network (working for a buyer)
Advertiser Ad Server
Demand-Side Platform
OpenRTB Client
Bidder Service
Sell-Side Platform (or Exchange)
OpenRTB Server
Auction Service
Ad Network (working for a pub)
Publisher Ad Server
Publisher
Data Management Platform
Data Broker

Key Points

1. DSP buys traffic for advertisers
2. Decision of displaying ad at the fixed platform is accepted by SSP via second price auction (lots of DSP's may take part)
3. DSP pays SSP the second price in case it won the auction and the ad was displayed at the platform

Monetization Policy

Advertiser pays DSP fixed price by one of the principle:

1. CPM (cost per mille)
   for each displaying of the ad at the platform (impression)
2. CPC (cost per click)
   for each user's click by the ad
3. CPA (cost per action)
   for each specified user's action (install of mobile application, finish some game level etc.)

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CPC and CPA – attempt to shift the part of the risk on the DSP.

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Advertisers risk:

1. Inappropriate market audience
2. Fraudulent traffic

DSP attempt of additional risk compensation – Dynamic Bidding Strategy.

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$\boxed{\textbf{An impression price depends on how its risky}}$

Recall $\{\xi_i, i \geq 1\}$ – ad traffic, $C \in \mathbb{N}$ – desired click volume.
The $ind(\xi_i)$ is an ad label: we assume $ind(\xi_i) = 1$ if the i-th request leaded to click, 0 – otherwise

$$N(C) = \min_n \left\{ n : \sum_{k=1}^{n} \mathbb{1}[ind(\xi_i) = 1] \geq C \right\}$$

B – advertiser's camp budget and maxCPC – maximal cost per click. Under condition traffic "may provide" clicks, holds:

$$\mathbb{E}\sum_{k=1}^{N(C)} maxCPC \cdot \mathbb{P}\left(ind(\xi_i) = 1\right) = B$$

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How to interpret this probability?

$$\mathbb{P}\left(ind(\xi_i) = 1\right) = \mathbb{P}\left(ind(\xi) = 1 \middle|\ \xi = x_i\right)$$

x_i – features characterise i-th ad bid

To sum up the “fair price” issue deduces to the posterior probability estimation of the positive class membership

$$\mathbb{P}\left(ind(\xi) = 1 \middle| \ \xi = x \right)$$

Problem Restrictions

1. High predictor computing speed
2. Active learning ability
3. High learning speed
4. Ability to parallel model training
5. High data dimension, high number of categoric features
6. Unbalanced training sample

Solutions

Method	Accuracy	Scalability	Efficiency
Bayes' naive	×	✓	✓
Logic trees	✓	×	×
Logistic	×	✓	✓
Logistic, hash-trick stochastic gradient	✓	✓	✓

Hashing Trick

$$h: \mathbb{N} \to \{1, ..., 2^b\}, \ \xi: \mathbb{N} \to \{-1, 1\}$$

$$\phi_i^{h,\xi}(x) = \sum_{j:\ h(j)=i} \xi(i) \cdot x_i, \ \ x \in \mathbb{R}^n$$

Hashing kernel:

$$(x, x^{'})_\phi \equiv \sum_{i=1}^{2^b} \phi_i^{h,\xi}(x) \cdot \phi_i^{h,\xi}(x^{'})$$

Under these notations holds:

$$\mathbb{E}(x, x^{'})_{\phi} = (x, x^{'})$$

If in addition $||x|| = ||x^{'}|| = 1$:

$$\mathbb{D}(x, x^{'})_{\phi} = O\left(\frac{1}{2^b}\right)$$

Stochastic gradient update:

$$w_{t+1} = w_t - \eta_t \cdot \nabla_w \ell\left(w_t^\intercal \cdot x_t, y_t\right)$$

Here

$$\ell(p, y) = \log(1 + e^{-yp}),\ \ y \in \{-1, 1\}$$

Converges condition:

$$\sum_{t=1}^{+\infty} \eta_t = \infty, \ \sum_{t=1}^{+\infty} \eta_t^2 < +\infty$$

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"Optimal" learning rate:

$$\eta_{t,i} = \frac{\alpha}{\beta + \sqrt{\sum_{s=1}^{t} g_{s,i}^2}}$$

$$g_s = \ell\left(w_s^{\mathsf{T}} \cdot x_s, y_s\right)$$

$$w_{t+1} = w_t - s(h) \cdot x,\ \ h \in \mathbb{R}^+$$

For logistic loss function:

$$s(h) = \frac{W(e^{h\eta \cdot x^\intercal x + yp + e^{yp}}) - h\eta \cdot x^\intercal x - e^{yp}}{y \cdot x^\intercal x}$$

Here $W(z)$ – Lambert function:

$$\forall z \in \mathbb{C} : z = W(z) \cdot e^{W(z)}$$

Model Valuation

Bidding model should provide an accurate estimation of the posterior probability. Over estimation would cause budget overruns. Under estimation would cause low conversion delivery rate.

- Classic approach:
  Interpret like binary classification model
- Typical metrics:
  auROC, Log-Loss, etc.
- Problem:
  We don't measure direct accuracy of probability estimators (classification thresholds)

Partition of the sample hashing space:

$$\mathfrak{X} = \bigsqcup_{j=1}^{d} O_j$$

Estimating accuracy of aggregated probabilities

$$\mathbb{P}\left(ind(\xi) = 1 \middle| \; \xi \in O_j \right)$$

might be performed via testing hypothesis of coherence theoretic and empiric distributions:

$$\mathbb{P}\left(\xi \in O_j \middle| \; ind(\xi) = 1 \right)$$

Solution: use Pearson's chi-square modified test

$$H_0 : P_j = P_t^{(j)}, \ j = \overline{1..d}$$

$$H_1 : P_j = P_{const}, \ j = \overline{1..d}$$

Chi-square statistic:

$$\chi^2_{d-1} = m \cdot \sum_{j=1}^{d} \frac{\left[P_t^{(j)} - P_e^{(j)}\right]^2}{P_t^{(j)}}$$

$$m = \left|\{i : ind(\xi_i) = 1\}\right|$$

Distribution of the chi-square statistic doesn't depend on the way of sample partition.

Efficiency criterion statistic:

$$\Psi_{d-1} = \frac{\chi^2_{d-1}(P_t^{(\cdot)})}{\chi^2_{d-1}(P_{const})}$$

Under consistency of the chi-square criterion holds asymptotic behaviour:

$$H_0 : \Psi_{d-1} \longrightarrow 0, \;\; m \to +\infty$$

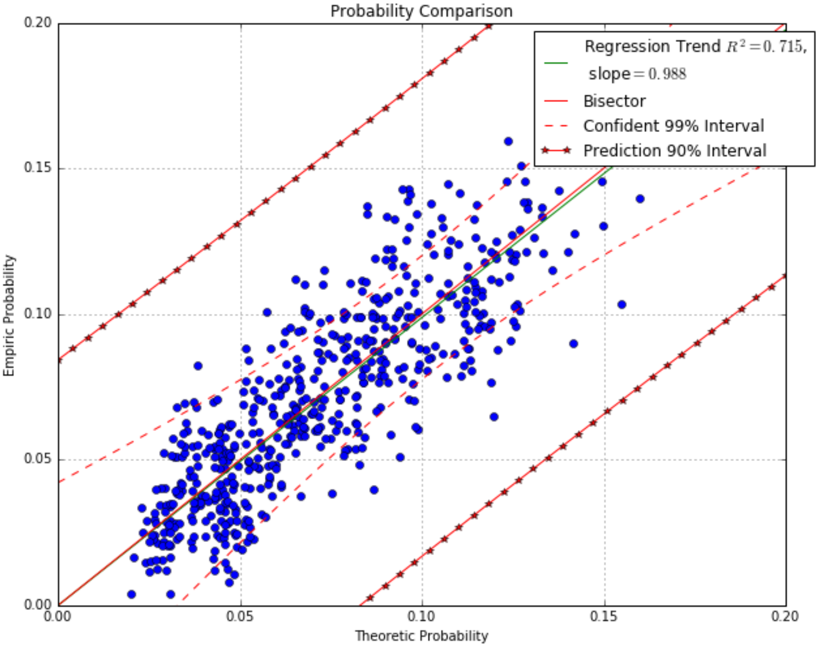
$$H_1 : \Psi_{d-1} \longrightarrow +\infty, \;\; m \to +\infty$$

If $\Psi_{d-1} < 1$ the dynamic model is better than static.

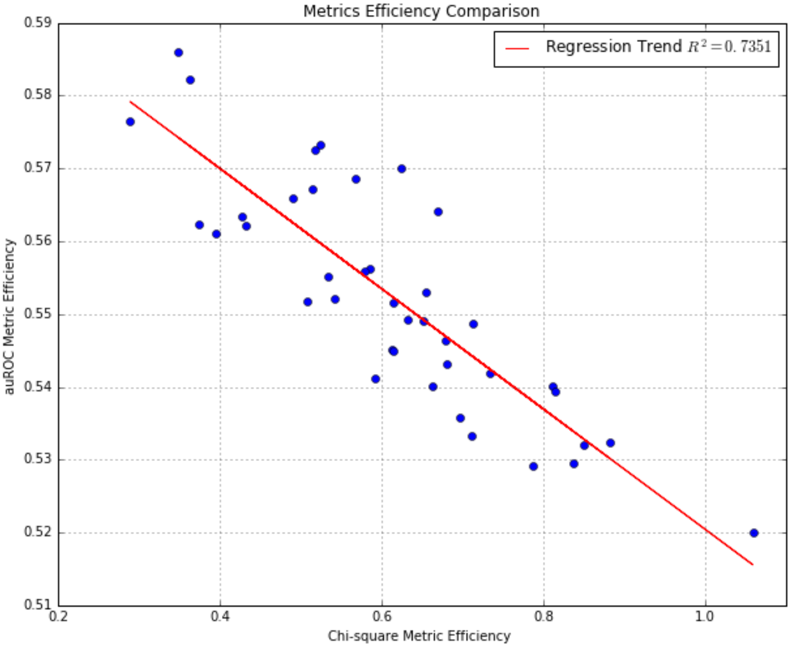
Based on QupleTech bidding history it was educated 40 logistic regression models by stochastic gradient descent with adaptive learning rate and hashing-trick features decoding by the principle:

- 80% training set
- 10% validation set
- 10% testing set

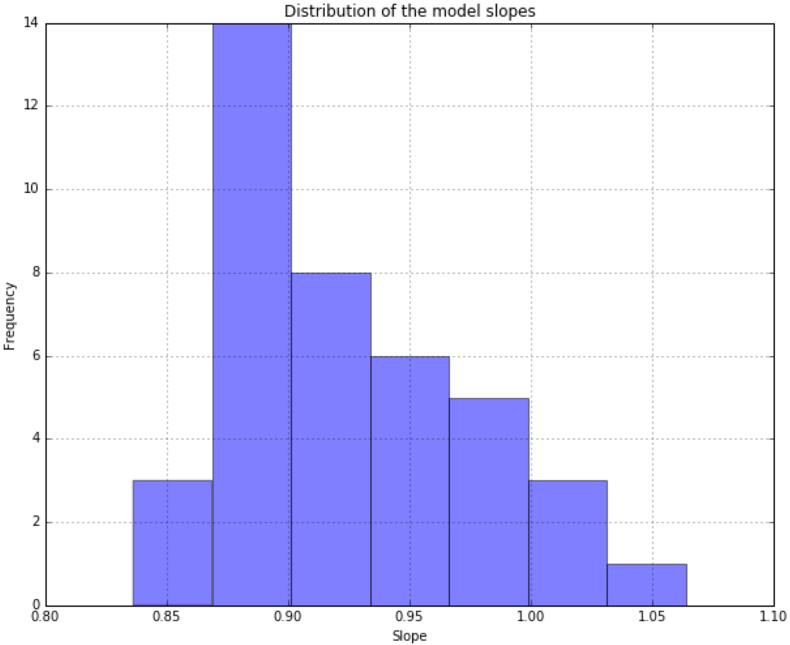
It were calculated aggregated model (theoretic) and observed (empiric) probability measures for each learning model by correspondent testing set.



Probability Comparison
Regression Trend $R^2 = 0.715$, slope $= 0.988$
Bisector
Confident 99% Interval
Prediction 90% Interval
Empiric Probability
Theoretic Probability
0.00
0.05
0.10
0.15
0.20



Metrics Efficiency Comparison
Regression Trend $R^2 = 0.7351$
auROC Metric Efficiency
Chi-square Metric Efficiency
0.59
0.58
0.57
0.56
0.55
0.54
0.53
0.52
0.51
0.2
0.4
0.6
0.8
1.0


Distribution of the model slopes
Frequency
Slope
0
2
4
6
8
10
12
14
0.80
0.85
0.90
0.95
1.00
1.05
1.10

Thank you for your attention!